the precursor decays by an amount

(81) $$\mathrm{d}u = \mathrm{d}p_x/\varrho_0 a_0 \,.$$

Under the assumed conditions, eq. (78) also applies along the path of the precursor. Combining eqs. (78) and (81) yields the relation

(82) $$\mathrm{d}p_x/\mathrm{d}t = -F/2 \,.$$

The function F is expected in general to be quite complicated. We can get a qualitative picture of its effect by assuming the form, for compression only,

(83) $$F = (p_x^e - p_x^s)/T \,, \qquad p_x^e > p_x^s,$$

where $T = \text{constant}$. Compression by the precursor is assumed to be elastic, so p_x of eq. (82) lies on a metastable extension of the elastic compression curve, $p_x^e(V)$. Above the yield point there is a stress $p_x^s(V)$ which will finally be reached for the given volume V after a very long time. This is curve AB of Fig. 14 *b*). According to eqs. (82) and (83), decay of the precursor amplitude, $p_x \equiv p_x^e(V)$ continues until $p_x^e(V) = p_x^s(V)$, which occurs at the static value of the Hugoniot elastic limit. To see the effect more explicitly, note that

(84) $$(\mathrm{d}/\mathrm{d}t)(p_x^e - p_x^s) = (1 - c^2/a^2)(\mathrm{d}p_x^e/\mathrm{d}t) \,,$$

where $c^2 = K/\varrho$, $a^2 = (K + 2\mu/3)/\varrho$. If Poisson's ratio, ν, is independent of density, so is c^2/a^2. Then eqs. (82)-(84) can be integrated to yield

(85) $$p_x^e(V) - p_x^s(V) = (p_x^e - p_x^s)_0 \exp\left[-x/x_0\right] ,$$

where

(86) $$x_0 = 2TD/(1 - c^2/a^2) \,.$$

Integrating eq. (84) under the assumption that $\nu = \text{constant}$ enables us to simplify eq. (85):

(87) $$p_x^e - p_{\mathrm{HEL}}^s = (p_x^e - p_{\mathrm{HEL}}^s)_0 \exp\left[-x/x_0\right] ,$$

where p_{HEL}^s is the static value of the Hugoniot elastic limit, related to the static yield strength by eq. (47).

Equation (82) was derived on the assumptions that the precursor follows a characteristic and that the energy equation, eq. (3), does not affect the propagation process. A more rigorous expression can be obtained by combining eq. (77) with eqs. (1)-(3) and specializing the result along the shock path [8]:

$$\frac{\mathrm{D}p_x}{\mathrm{D}x} = \left(1 - \frac{u}{D}\right) \frac{(D-u)^2 - a^2}{\frac{3}{2}(D-u)^2 + a^2/2} \frac{\partial p_x}{\partial \mathrm{x}} - \frac{(D-u)^2}{D} \frac{F'}{\frac{3}{2}(D-u)^2 + a^2/2}, \tag{88}$$

$$F' = (1 - \alpha \Gamma y/2\mu)F. \tag{89}$$

Here the block derivative, $\mathrm{D}/\mathrm{D}x$, refers to differentiation along the shock path, $\partial p_x/\partial x$ is evaluated immediately behind the precursor front, and F' is a modification to F resulting from the assumption that a fraction α of plastic work goes into heat. In eq. (89), Γ is the Gruneisen parameter. F' and F differ by less than 10% for metals in which plastic flow occurs.

Under the assumptions that $D-u=a$ and $\alpha=0$, eq. (88) reduces to eq. (82).

Considerable effort in recent years has been devoted to attempts to relate the relaxation function F of eq. (75) to the motion and multiplication of dislocations. The basic relation is

$$\mathrm{d}E^p/\mathrm{d}t = hNbv = F/2\mu, \tag{90}$$

where N is the number of dislocations per unit area, b is the Burgers vector, h is a numerical constant the order of units, and v is the mean velocity of dislocations. Since $E_p = 2\varepsilon_1/3$ in uniaxial strain, eq. (90) becomes

$$\mathrm{d}\varepsilon_1/\mathrm{d}t = 3hNbv/2. \tag{91}$$

There are various models for multiplication and motion of dislocations. One which is frequently used is due to Gilman:

$$N = N_{om}(1 + A\varepsilon^p), \tag{92}$$

$$v = v_{\max} \exp[-D/\tau], \tag{93}$$

where

N_{om} = initial density of mobile dislocations,

$v_{\max}$ = maximum dislocation velocity $\sim v_{\text{shear}}$,

D = drag coefficient,

A = multiplication coefficient,

τ = resolved shear stress $= (p_x - p_y)/2$.